

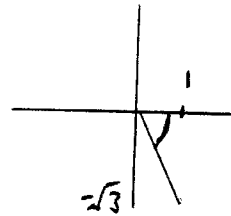
2a)

$$i) \quad w = 3e^{-\frac{\pi}{12}j}$$

$$z = 1 - \sqrt{3}j$$

$$|w| = 3$$

$$\arg(w) = -\frac{\pi}{12}$$



$$\tan^{-1} \frac{\sqrt{3}}{1} = \frac{\pi}{3}$$

$$|z| = \sqrt{1^2 + (-\sqrt{3})^2} = 2$$

$$\arg(z) = -\frac{\pi}{3}$$

$$\left| \frac{w}{z} \right| = \frac{3}{2}$$

$$\arg\left(\frac{w}{z}\right) = -\frac{\pi}{12} - \left(-\frac{\pi}{3}\right)$$

$$= -\frac{\pi}{12} + \frac{4\pi}{12} = \frac{3\pi}{12} = \frac{\pi}{4}$$

$$\arg\left(\frac{w}{z}\right) = \frac{\pi}{4}$$

ii)

$$\frac{w}{z} = \frac{3}{2} e^{j\frac{\pi}{4}} = \frac{3}{2} \left(\cos \frac{\pi}{4} + j \sin \frac{\pi}{4} \right)$$

$$= \frac{3}{2\sqrt{2}} + \frac{3}{2\sqrt{2}} j$$

2b)

$$i) \quad e^{-\frac{1}{2}ja} + e^{\frac{1}{2}ja} = 2 \cos\left(\frac{a}{2}\right)$$

$$1 + e^{ja} = e^{\frac{1}{2}ja} \left(e^{-\frac{1}{2}ja} + e^{\frac{1}{2}ja} \right)$$

$$= e^{\frac{1}{2}ja} \left(2 \cos\left(\frac{a}{2}\right) \right) = 2 e^{\frac{1}{2}ja} \cos\left(\frac{a}{2}\right)$$

2b ii)

$$C = 1 + \binom{n}{1} \cos \theta + \binom{n}{2} \cos 2\theta + \dots + \binom{n}{n} \cos n\theta$$

$$S = \binom{n}{1} \sin \theta + \binom{n}{2} \sin 2\theta + \dots + \binom{n}{n} \sin n\theta$$

$$C + jS = 1 + \binom{n}{1} (\cos \theta + j \sin \theta) + \binom{n}{2} (\cos 2\theta + j \sin 2\theta) + \dots + \binom{n}{n} (\cos n\theta + j \sin n\theta)$$

$$C + jS = 1 + \binom{n}{1} e^{j\theta} + \binom{n}{2} e^{j2\theta} + \dots + \binom{n}{n} e^{jn\theta}$$

$$C + jS = (1 + e^{j\theta})^n$$

$$\begin{aligned} \Rightarrow C + jS &= \left(2 e^{j\frac{\theta}{2}} \cos\left(\frac{\theta}{2}\right) \right)^n \\ &= 2^n e^{j\frac{n\theta}{2}} \cos^n\left(\frac{\theta}{2}\right) = 2^n \left(\cos\left(\frac{n\theta}{2}\right) + j \sin\left(\frac{n\theta}{2}\right) \right) \cos^n\left(\frac{\theta}{2}\right) \end{aligned}$$

Equating real and imaginary parts

$$C = 2^n \cos\left(\frac{n\theta}{2}\right) \cos^n\left(\frac{\theta}{2}\right)$$

$$S = 2^n \sin\left(\frac{n\theta}{2}\right) \cos^n\left(\frac{\theta}{2}\right)$$

$$\frac{S}{C} = \frac{2^n \sin\left(\frac{n\theta}{2}\right) \cos^n\left(\frac{\theta}{2}\right)}{2^n \cos\left(\frac{n\theta}{2}\right) \cos^n\left(\frac{\theta}{2}\right)} = \tan\left(\frac{n\theta}{2}\right)$$