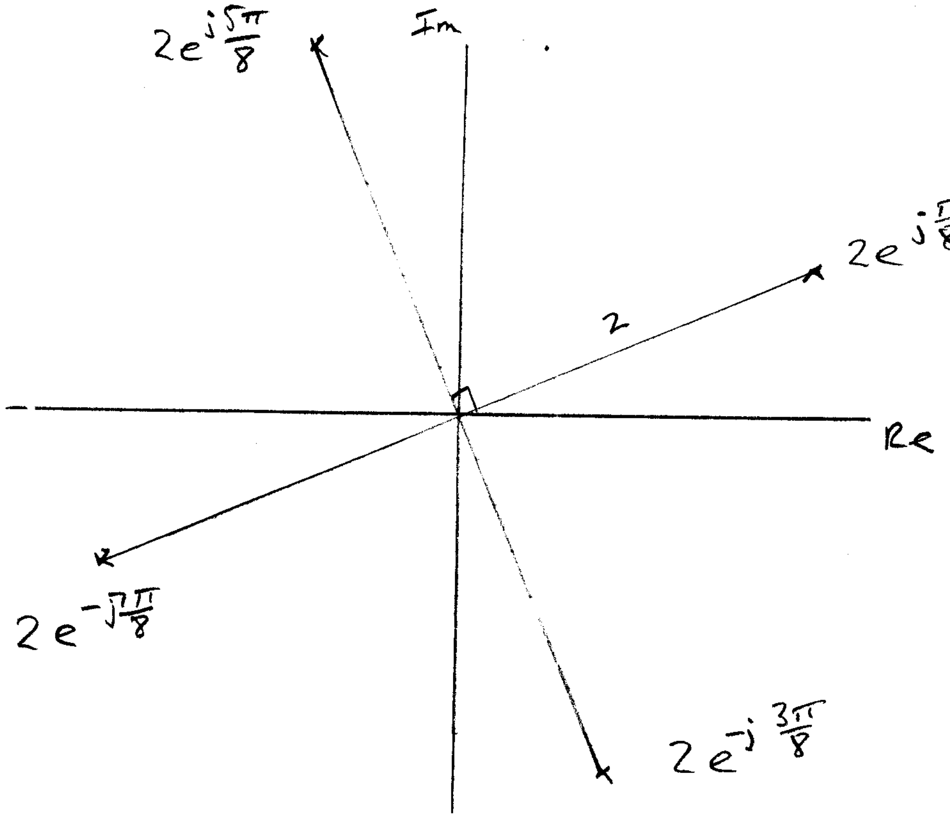
FP2 JANUARY 2008 Q2 COMPLEX NUMBERS

2a) $16j = 16e^{j\frac{\pi}{2}}$

4^{th} roots given by $16^{\frac{1}{4}} e^{j\left(\frac{\pi}{8}+\frac{2n\pi}{4}\right)}$ for $n=0,1,2,3$

$= 2e^{j\frac{\pi}{8}},\ 2e^{j\frac{5\pi}{8}},\ 2e^{-j\frac{7\pi}{8}},\ 2e^{-j\frac{3\pi}{8}}$



2b)

i) $(1-2e^{j\theta})(1-2e^{-j\theta})$

$= 1 - 2e^{j\theta} - 2e^{-j\theta} + 4$

$= 5 - 2(e^{j\theta} + e^{-j\theta})$

$= 5 - 2(2\cos\theta)$

$= 5 - 4\cos\theta$

2b)

ii) $C = 2\cos\theta + 4\cos 2\theta + 8\cos 3\theta + \dots + 2^n \cos n\theta$

$S = 2\sin\theta + 4\sin 2\theta + 8\sin 3\theta + \dots + 2^n \sin n\theta$

$C + jS = 2(\cos\theta + j\sin\theta) + 4(\cos 2\theta + j\sin 2\theta) + \dots + 2^n(\cos n\theta + j\sin n\theta)$

$C + jS = 2e^{j\theta} + 4e^{j2\theta} + 8e^{j3\theta} + \dots + 2^n e^{jn\theta}$

This is a GP with first term $a = 2e^{j\theta}$

common ratio $r = 2e^{j\theta}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$C + jS = \frac{2e^{j\theta}(1 - 2^n e^{jn\theta})}{1 - 2e^{j\theta}}$$

$$\Rightarrow C + jS = \frac{2e^{j\theta}(1 - 2^n e^{jn\theta})(1 - 2e^{-j\theta})}{(1 - 2e^{j\theta})(1 - 2e^{-j\theta})}$$

$$\Rightarrow C + jS = \frac{2e^{j\theta}(1 - 2^n e^{jn\theta} - 2e^{-j\theta} + 2^{n+1} e^{j(n-1)\theta})}{5 - 4\cos\theta}$$

$$\Rightarrow C + jS = \frac{2e^{j\theta} - 2^{n+1} e^{j(n+1)\theta} - 4 + 2^{n+2} e^{jn\theta}}{5 - 4\cos\theta}$$

$$\Rightarrow C + jS = \frac{2(\cos\theta + j\sin\theta) - 2^{n+1}(\cos(n+1)\theta + j\sin(n+1)\theta) - 4 + 2^{n+2}(\cos n\theta + j\sin n\theta)}{5 - 4\cos\theta}$$

2biii cont) Equating real and imaginary parts

$$C = \frac{2\cos\theta - 2^{n+1}\cos(n+1)\theta - 4 + 2^{n+2}\cos n\theta}{5 - 4\cos\theta}$$

$$S = \frac{2\sin\theta - 2^{n+1}\sin(n+1)\theta + 2^{n+2}\sin n\theta}{5 - 4\cos\theta}$$