OCR
RECOGNISING ACHIEVEMENT

General Certificate of Education
Advanced Supplementary (AS) and Advanced Level
former Oxford and Cambridge modular syllabus

MEI STRUCTURED MATHEMATICS 5509

Mechanics 3

Monday 22 JANUARY 2001 Afternoon 1 hour 20 minutes

Additional materials:
Answer paper
Graph paper
Students' Handbook

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** questions.

Write your answers on the separate answer paper provided.

If you use more than one sheet of paper, fasten the sheets together.

INFORMATION FOR CANDIDATES

The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.

You are advised that an answer may receive no marks unless sufficient detail of the working is shown on the answer paper to indicate that a correct method is being used.

Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.

This question paper consists of 5 printed pages and 3 blank pages.

1 An experiment consists of stretching a length of wire and fixing its ends. The wire is then plucked to produce transverse vibrations. The frequency of these vibrations, f, is modelled by the equation

$$f = kP^{\alpha}l^{\beta}z^{\gamma}$$

where P is the tension in the wire, l is the length of the stretched wire, z is the mass per unit length of the stretched wire and k is a dimensionless constant.

(i) Given that the dimensions of frequency are T^{-1}, use dimensional analysis to show that $\alpha = \frac{1}{2}$, $\beta = -1$ and $\gamma = -\frac{1}{2}$. [5]

Two pieces of wire, A and B, used in the experiment both have modulus of elasticity 500 N.

(ii) Piece A has natural length 0.9 m and stretched length 0.945 m. Calculate the tension in the wire. [2]

(iii) Piece B is stretched to a length of 0.81 m. The tension in the wire is 40 N. Calculate the natural length of the wire. [4]

(iv) *In their unstretched state*, the two wires had the same mass per unit length. In the experiment, wires A and B produce frequencies f_A and f_B respectively. Calculate the ratio $f_A : f_B$. [4]

2 A pendulum consists of a light rod, AB, of length 0.8 m with a small brass sphere of mass 0.5 kg attached at the end B. The end A is freely pivoted so that the pendulum can move in a vertical plane. Initially the pendulum is held slightly displaced from the vertical and released from rest. The angle the pendulum makes with the vertical after t seconds is denoted by θ radians, as shown in Fig. 2.

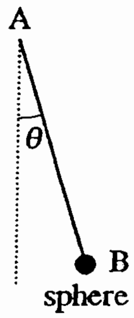


Fig. 2

(i) Draw a diagram to show the forces acting on the sphere after it has been released. [2]

(ii) Write down the equation of motion for the sphere in the tangential direction. Show that, if θ is small, it satisfies approximately the simple harmonic motion equation

$$\ddot{\theta} = -12.25\theta. \quad [5]$$

The pendulum is released with $\theta = 0.1$.

(iii) Write down the amplitude and period of the motion. [2]

(iv) Calculate the time at which θ is first equal to 0.05. [3]

(v) Calculate the positive value of θ for which the magnitude of the angular velocity of the pendulum is half of its maximum value. [3]

[Turn over

3 Fig 3.1 shows a uniform lamina OAB in the shape of the region between the curve $y = kx^{\frac{3}{2}}$, the x-axis and the line $x = 1$, where k is a positive constant.

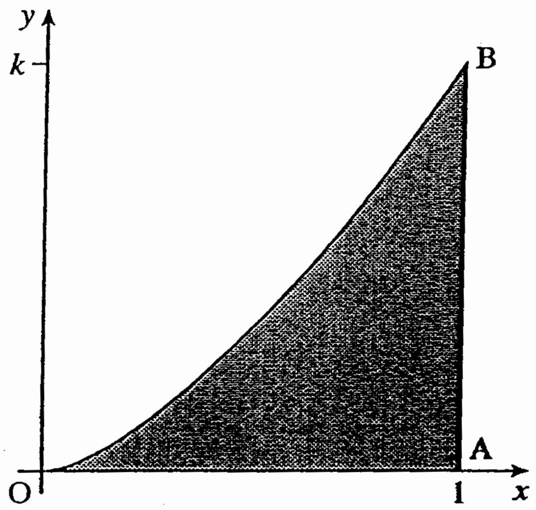


Fig. 3.1

(i) Show, by integration, that the coordinates of the centre of mass of the lamina are $\left(\frac{5}{7}, \frac{5}{16}k\right)$. [9]

The lamina is suspended from the vertex A and hangs in equilibrium as shown in Fig. 3.2. The angle that AB makes with the vertical is denoted by θ.

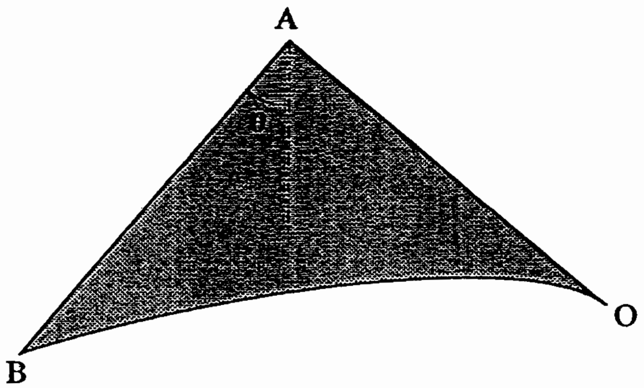


Fig. 3.2

(ii) Find $\tan\theta$ in terms of k. [2]

(iii) Calculate the value of k for which O is at the same horizontal level as B. [4]

4 A car of mass 600 kg comes to a bend in a road. While travelling round the bend, the car moves in an arc of a horizontal circle of radius 40 m. As the car travels round the bend there is a frictional force perpendicular to the direction of motion. The forces acting on the car in the direction of motion may be neglected.

The road surface is horizontal, as shown in Fig. 4.1.

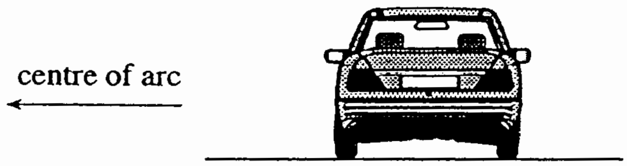


Fig. 4.1

(i) Calculate the frictional force on the car when it travels round the bend at a speed of $15\ \mathrm{m\,s^{-1}}$. [3]

The coefficient of friction between the car and the road is 0.8.

(ii) Calculate the maximum speed at which the car can travel round the bend without slipping sideways. [3]

The road is to be redeveloped and is to be banked at an angle θ to the horizontal, as shown in Fig. 4.2. The coefficient of friction is still 0.8.

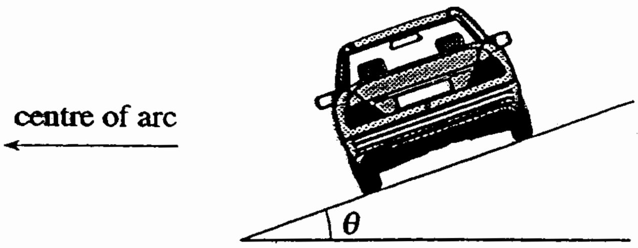


Fig. 4.2

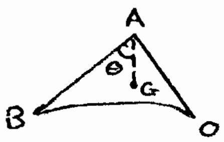
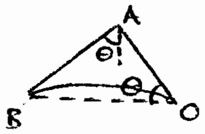
The car is driven round the bend at the maximum safe speed, i.e. the speed at which it is on the point of slipping up the slope.

(iii) In the case where $\theta = 4°$, write down the vertical equilibrium equation and calculate the normal reaction between the road and the car. Calculate the maximum safe speed of the car in this case. [5]

(iv) Calculate the value of θ which would give a maximum safe speed of $25\ \mathrm{m\,s^{-1}}$. [4]

Mark Scheme

1. (i)	$T^{-1} = (MLT^{-2})^{\alpha}L^{\beta}(ML^{-1})^{\gamma}$	M1 A1	
	$0 = \alpha + \gamma$		
	$0 = \alpha + \beta - \gamma$	M1	two equations
	$-1 = -2\alpha$	M1	third equation
	$\alpha = \frac{1}{2},\ \beta = -1,\ \gamma = -\frac{1}{2}$	E1	
(ii)	$T = \frac{500\times0.045}{0.9}$	M1	Hooke's law
	$= 25$ N	A1	
(iii)	$40 = \frac{500(0.81-l_0)}{l_0}$	M1 A1	equation
	$40l_0 = 405 - 500l_0$	M1	solving
	$l_0 = 0.75$	A1	
(iv)	$\frac{f_A}{f_B} = \left(\frac{T_A}{T_B}\right)^{\frac{1}{2}}\left(\frac{l_A}{l_B}\right)^{-1}\left(\frac{z_A}{z_B}\right)^{-\frac{1}{2}}$	M1	using formula for f_A or f_B
	$= \left(\frac{25}{40}\right)^{\frac{1}{2}}\left(\frac{0.945}{0.81}\right)^{-1}\left(\frac{0.9m/0.945}{0.75m/0.81}\right)^{-\frac{1}{2}}$	M1	attempt ratio
		M1	0.75/0.81 or 0.9/0.945 or equivalent seen
	= 0.668 so ratio 0.668:1 or 1:1.497	A1	or any equivalent form
2. (i)	Tension, weight	B1 B1	no spurious forces
(ii)	$0.5.0.8\ddot{\theta} = -0.5g\sin\theta$	M1	N2L
		A1	LHS
		A1	RHS
	$\ddot{\theta} = -12.25\sin\theta$		
	$\sin\theta \approx \theta \Rightarrow \ddot{\theta} = -12.25\theta$	M1 E1	
(iii)	$a = 0.1$	B1	
	$T = \frac{2\pi}{\sqrt{12.25}} = \frac{4\pi}{7}$ (= 1.795)	B1	
(iv)	$\theta = 0.1\cos 3.5t$	B1	
	$0.05 = 0.1\cos 3.5t$	M1	$\theta = 0.05$ and solve
	$t = \frac{2\pi}{21} = 0.299$	A1	
(v)	max. ang. vel. $= 0.1\times3.5 = 0.35$	M1	
	$\left(\frac{0.35}{2}\right)^2 = 3.5^2(0.1^2 - \theta)^2$	M1	
	[or $\dot{\theta} = \frac{0.35}{2} \Rightarrow t = \frac{\pi}{21} = 1.496$]		
	$\theta = \frac{\sqrt{3}}{20} = 0.0866$	A1	

3. (i) Area $= \int_0^1 kx^{\frac{3}{2}}dx = \left[\frac{2}{5}kx^{\frac{5}{2}}\right]_0^1 = \frac{2k}{5}$	M1 A1	
$\frac{2k}{5}\bar{x} = \int_0^1 x.kx^{\frac{3}{2}}dx$	M1	using $\int xy$
$= \left[\frac{2}{7}kx^{\frac{7}{2}}\right]_0^1$	M1	
$= \frac{2k}{7}$		
$\bar{x} = \frac{5}{7}$	M1	divide by area (for either co-ordinate)
	E1	
$\frac{2k}{5}\bar{y} = \int_0^1 \frac{1}{2}\left(kx^{\frac{3}{2}}\right)^2 dx$	M1	using $\int \frac{1}{2}y^2$
$= \left[\frac{k^2x^4}{8}\right]_0^1$	M1	
$= \frac{k^2}{8}$		
$\bar{y} = \frac{5k}{16}$	E1	
(ii)  $\tan\theta = \frac{1-\frac{5}{7}}{\frac{5k}{16}}$	M1	
$= \frac{32}{35k}$	A1	
(iii)  $<AOB = \theta$	M1	attempt at angle in terms of θ
$\Rightarrow \tan\theta = \frac{k}{1} = k$	A1	
$\frac{32}{35k} = k \Rightarrow k = \sqrt{\frac{32}{35}}$	M1 A1	
4. (i) $F = \frac{mv^2}{r}$	M1	N2L
$= \frac{600.15^2}{40}$	M1	acceleration
$= 3375$ N	A1	
(ii) $R = 600g$		
$F = \mu R = 0.8.600g$	M1	
$0.8.600g = \frac{600v^2}{40}$	M1	N2L
$v = 17.7$ m s^{-1}	A1	
(iii) $R\cos 4° - 0.8R\sin 4° = 600g$	M1	vertical equation with cpt of R
$R = 6244$ N	A1	
$R\sin 4° + 0.8R\cos 4° = \frac{600v^2}{40}$	M1	radial equation with friction and $\frac{mv^2}{r}$ (must resolve at least one term)
	A1	
$v = 19.0$ m s^{-1}	A1	
(iv) $R\cos\theta - 0.8R\sin\theta = 600g$	M1	vertical equation (attempt all terms)
$R\sin\theta + 0.8R\cos\theta = \frac{600\times 25^2}{40}$	M1	radial equation (attempt all terms)
$\frac{\sin\theta+0.8\cos\theta}{\cos\theta-0.8\sin\theta} = \frac{25^2}{40g}$	M1	eliminate R and attempt to solve
$892\sin\theta = 311.4\cos\theta$		
$\tan\theta = 0.349$		
$\theta = 19.2°$	A1	

Examiner’s Report

Mechanics 3 (5509)

General Comments

The responses to this paper often showed a reasonable level of competence in mechanics. However there were common weaknesses shown in maintaining accuracy in calculations. A number of candidates also seemed unfamiliar with some standard topics, such as the simple pendulum and motion on a banked track. Again, candidates must be reminded to show sufficient working to justify answers given in the question.

Comments on Individual Questions

Question 1 (Elastic wire)

This was generally the best answered question with many high marks. The first part was usually answered very well, although a few candidates gave no equations and just stated the given answers without any real justification. The second and third parts were generally done well. The calculation of the ratio in the last part often caused problems. The most common error by far was to assume the mass per unit length was the same in the stretched state, contrary to the given information.

(ii) 25 N, (iii) 0.75 m, (iv) 0.668:1.

Question 2 (Simple pendulum)

The force diagram was usually done well, but few candidates were able to use it to write down a correct Newton's second law equation. Equations often involved the tension or the wrong component of weight. Few candidates used the correct acceleration, and some wrote down equations for a conical pendulum. However, many candidates were then able to use the given equation to make progress with the remaining parts of the question. These were often done well, however some candidates were unable to correctly identify the amplitude and the expression for θ was often given as a sine rather than cosine function. In the last part of the question, some candidates were unable to calculate the maximum angular velocity, and others confused this with the symbol ω used in the standard SHM equation.

(iii) 0.1, 1.80 s, (iv) 0.299 s, (v) 0.0866.

Question 3 (Centre of mass)

Some candidates could make little progress with the calculation of the centre of mass as they did not know the correct formulae to use – indeed some used the volume of revolution formula. However many knew the formula for the x-coordinate. The y-coordinate caused more problems with fewer knowing the formula. The calculation of $\tan\theta$ was often hampered by assuming that it was merely the ratio of the coordinates given in (i). Those who drew diagrams were often more successful in identifying the relevant lengths, although the ratio was sometimes calculated the wrong way or hampered by poor manipulation of fractions. The last part of the question was often wrong as candidates commonly assumed that θ was 45°.

(ii) $\frac{32}{35k}$, (iii) $\sqrt{\frac{32}{35}}$.

Question 4 (Circular motion)

The first two parts of this question were generally well done. The last two parts were generally done poorly. The attempts were hampered by poor diagrams and poor equations of motion. Candidates often omitted forces and/or failed to resolve correctly.

(i) 3375 N, (ii) 17.7 $\mathrm{m\,s^{-1}}$, (iii) 19.0 $\mathrm{m\,s^{-1}}$, (iv) 19.2°.