OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education**
**Advanced General Certificate of Education**

# MEI STRUCTURED MATHEMATICS 2609

Mechanics 3

Friday **14 JUNE 2002** Morning 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

**TIME** 1 hour 20 minutes

## INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer all questions.
- You are permitted to use a graphical calculator in this paper.

## INFORMATION FOR CANDIDATES

- The approximate allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8$ m s$^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

**This question paper consists of 5 printed pages and 3 blank pages.**



[Turn over

1 **(i)** Write down the dimensions of velocity, acceleration and force. [2]

**(ii)** Use the definitions of work, kinetic energy and change in gravitational potential energy to show that these quantities have the same dimensions. [3]

The tension in a stretched wire is given by $T = \frac{YAx}{l_0}$, where $A$ is the cross-sectional area of the wire, $l_0$ is the natural length of the wire, $x$ is the extension and $Y$ is a quantity called Young's modulus which depends on the material from which the wire is made.

**(iii)** Determine the dimensions of Young's modulus. [3]

The energy stored in the stretched wire is given by the equation

$$E = cY^{\alpha}\left(\frac{A}{l_0}\right)^{\beta} x^{\gamma}$$

where $c$ is a dimensionless constant.

**(iv)** Use dimensional analysis to determine the value of $\alpha$ and to find a relationship between $\beta$ and $\gamma$. [4]

**(v)** Use the standard formulae for tension and energy in terms of stiffness and extension to determine the values of $\beta$, $\gamma$ and the constant $c$. [3]

2 A weighing machine is being designed. It consists of a square platform of mass 2.5 kg supported by a number of identical springs each of stiffness $25\,000\,\mathrm{N\,m^{-1}}$, which are attached to a fixed horizontal base as shown in Fig. 2.

Throughout this question assume that the platform remains horizontal.

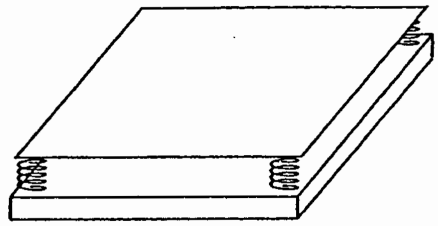

**Fig. 2**

Initially the designer uses four springs and the system is in equilibrium.

**(i)** Calculate the compression in each spring before any object is placed on the platform. [2]

A child of mass 30 kg is standing on the platform, which is at rest.

**(ii)** Calculate the compression in each of the four springs. [2]

**(iii)** Calculate the minimum number of additional springs required to reduce the compression to less than 0.002 m. [4]

The 30 kg child is standing on the platform supported by four springs as in the original design. The child's father lifts her off quickly, allowing the platform to oscillate freely in a vertical direction.

**(iv)** The displacement of the platform *below* the equilibrium position at time $t$ seconds is $y$ metres. Write down the equation of motion for the platform. Hence show that the platform performs simple harmonic motion of period $\frac{1}{100}\pi$ s. Calculate the maximum speed of the platform. [7]

3 Fig. 3.1 shows a conical pendulum. A light inextensible chain AB of length 2 m is fixed at the end A and a small object of mass $m$ kg is attached at the end B. The object is rotating in a horizontal circle with constant angular speed $\omega$ rad s$^{-1}$. The chain makes an angle $\alpha$ with the vertical throughout the motion.

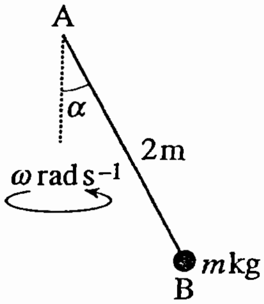


**Fig. 3.1**

**(i)** The tension in the chain is $T$ N. Express $T$ in terms of $m$ and $\omega$, and show that

$$\cos\alpha = \frac{g}{2\omega^2}.$$ [6]

A children's fairground ride is modelled as a rigid horizontal bar OA of length 2 metres and the chain AB from part **(i)**. The end O of the bar is fixed to an axle rotating with constant angular speed $\omega$ rad s$^{-1}$. The chain makes an angle $\beta$ with the vertical, as shown in Fig. 3.2. The object of mass $m$ kg is attached at B as before.

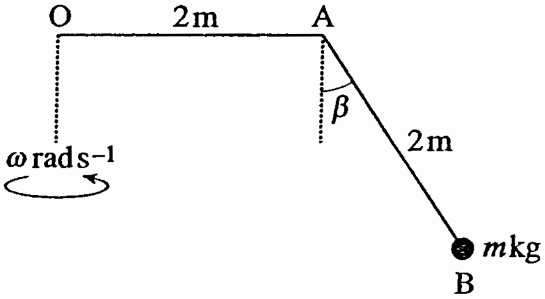


**Fig. 3.2**

**(ii)** Write down the vertical and radial equations for the motion of the object. [4]

**(iii)** Use your equations in part **(ii)** to calculate the angular speed when $\beta = 30°$. [3]

**(iv)** Show that if the ride rotates at 0.7 rad s$^{-1}$, then $\beta$ satisfies the equation

$$10\tan\beta - \sin\beta = 1.$$ [2]

4 A uniform solid hemisphere of radius $r$ is formed by rotating the region in the first quadrant within the curve $x^2 + y^2 = r^2$ through $2\pi$ radians about the $x$-axis, as shown in Fig. 4.1.

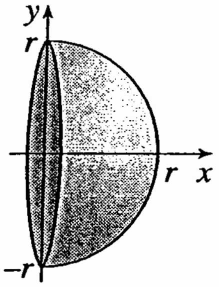


**Fig. 4.1**

**(i)** Find, by integration, the volume of the hemisphere and show that the centre of mass of the hemisphere has coordinates $(\frac{3}{8}r, 0)$. [6]

A hemisphere of radius $kr$ (where $0 < k < 1$) is removed from a hemisphere of radius $r$ to leave a uniform hemispherical shell of constant thickness, as shown in cross-section in Fig. 4.2.

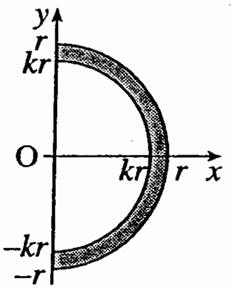


**Fig. 4.2**

**(ii)** Show that the $x$-coordinate of the centre of mass of the shell is $\frac{3}{8}r\left(\frac{1-k^4}{1-k^3}\right)$. [4]

**(iii)** By writing $k = 1 - \varepsilon$ where $\varepsilon$ is small, show that $1 - k^3 \approx 3\varepsilon$. Find a similar expression for $1 - k^4$. Hence, or otherwise, show that the centre of mass of a hemispherical shell of negligible thickness is at the midpoint of the axis of symmetry of the shell. [5]

# Mark Scheme

| | | | | |
|---|---|---|---|---|
| 1(i) | [velocity] = $LT^{-1}$ [acceleration] = $LT^{-2}$ | B1 | | |
| | [force] = $MLT^{-2}$ | B1 | | 2 |
| (ii) | [work done] = [F.d] = $MLT^{-2}.L = ML^2T^{-2}$ | B1 | Must be shown, not just stated | |
| | [KE] = [½ $mv^2$] = $M(LT^{-1})^2 = ML^2T^{-2}$ | B1 | | |
| | [GPE] = [$mgh$] = $M.LT^{-2}.L = ML^2T^{-2}$ | B1 | | 3 |
| (iii) | $Y = \frac{Tl_0}{Ax}$ | M1 | rearranging | |
| | $[Y] = \frac{MLT^{-2}.L}{L^2.L}$ | M1 | substitute dimensions | |
| | $= ML^{-1}T^{-2}$ | A1 | cao | 3 |
| (iv) | $ML^2T^{-2} = (ML^{-1}T^{-2})^{\alpha}L^{\beta}L^{\gamma}$ | M1 | substitute dimensions | |
| | $\alpha = 1$ | A1 | | |
| | $-1 + \beta + \gamma = 2$ | M1 | equating powers of L | |
| | $\beta + \gamma = 3$ | A1 | cao | 4 |
| (v) | $T = kx \Rightarrow k = \frac{YA}{l_0}$ | M1 | use tension to relate $Y$ to $k$ or $\lambda$ | |
| | $E = \frac{1}{2}kx^2 = \frac{1}{2}\frac{YA}{l_0}x^2$ | M1 | use energy to deduce at least two of $\beta$, $\gamma$, $c$ | |
| | $\Rightarrow \beta = 1, \gamma = 2, c = ½$ | A1 | | 3 |

| | | | | |
|---|---|---|---|---|
| 2(i) | $4(25\,000x) = 2.5g$ | M1 | use of Hooke's law | |
| | $x = 0.000\,245$ m | A1 | cao | 2 |
| (ii) | $4(25000x) = 32.5g$ | M1 | use of Hooke's law with new mass | |
| | $x = 0.003\,185$ m | A1 | cao | 2 |
| (iii) | $n(25\,000x) = 32.5g$ | M1 | equilibrium equation involving $n$ (or equivalent) | |
| | $x < 0.02 \Rightarrow n > 6.37$ | M1 | solving | |
| | | A1 | 6.37 | |
| | $\Rightarrow$ min. number of extra springs is 3 | A1 | cao | 4 |
| (iv) | $2.5\ddot{y} = 2.5g - 4 \times 25000(y + 0.000245)$ | M1 | N2L | |
| | | M1 | linear expression in $y$ for force in spring | |
| | | M1 | a weight term | |
| | | A1 | all correct with consistent signs | |
| | $\Rightarrow \ddot{y} = -40000y \Rightarrow$ SHM | E1 | must be all correct | |
| | period = $2\pi/200 = \pi/100$ | E1 | | |
| | amp. = 0.003185 – 0.000245 = 0.00294 | | | |
| | max. speed = 0.00294×200 = 0.588 m $s^{-1}$ | B1 | cao | 7 |

| | | | | |
|---|---|---|---|---|
| 3(i) | $T\cos\alpha = mg$ | B1 | | |
| | $T\sin\alpha = mr\omega^2$ | M1 | N2L with radial acceleration | |
| | $T\sin\alpha = m(2\sin\alpha)\omega^2$ | M1 | eliminate $r$ | |
| | $T = 2m\omega^2$ | A1 | | |
| | $\cos\alpha = \frac{mg}{T} = \frac{g}{2\omega^2}$ | M1<br>E1 | eliminate $T$ | |
| | | | | 6 |
| (ii) | $r = 2 + 2\sin\beta$ | B1 | | |
| | $T\cos\beta = mg$ | B1 | | |
| | $T\sin\beta = mr\omega^2$ | M1 | N2L with $mr\omega^2$ and component of tension | |
| | $= m(2+2\sin\beta)\omega^2$ | A1 | may be awarded later | |
| | | | | 4 |
| (iii) | $\beta = 30° \Rightarrow \frac{\sqrt{3}}{2}T = mg,\ \frac{1}{2}T = 3m\omega^2$ | M1 | substitute angle | |
| | $\Rightarrow \omega^2 = \frac{g}{3\sqrt{3}}$ | M1 | eliminate $T$ and use reasonable attempt at $r$ | |
| | $\omega = 1.37$ rad $s^{-1}$ | A1 | cao | |
| | | | | 3 |
| (iv) | $\tan\beta = \frac{\omega^2}{g}(2+2\sin\beta)$ | M1 | eliminate $T$ and use correct $r$ | |
| | $\omega = 0.7 \Rightarrow 10\tan\beta - \sin\beta = 1$ | E1 | | |
| | | | | 2 |

| | | | | |
|---|---|---|---|---|
| 4(i) | $V = \int_0^r \pi(r^2 - x^2)dx = \left[r^2x - \frac{1}{3}x^3\right]_0^r$ | M1 | use of formula and attempt to integrate | |
| | $= \frac{2}{3}\pi r^3$ | A1 | | |
| | $V\bar{x} = \int_0^r \pi x(r^2 - x^2)dx$ | M1 | use of formula and attempt to integrate | |
| | $= \pi\left[\frac{1}{2}r^2x^2 - \frac{1}{4}x^4\right]_0^r$ | A1 | | |
| | $\bar{x} = \frac{\frac{1}{4}\pi r^4}{\frac{2}{3}\pi r^3} = \frac{3}{8}r$ | E1 | | |
| | $\bar{y} = 0$ by symmetry | E1 | | |
| | | | | 6 |
| (ii) | $\bar{x} = \frac{\frac{3}{8}r\cdot\frac{2}{3}\pi r^3 - \frac{3}{8}kr\cdot\frac{2}{3}\pi(kr)^3}{\frac{2}{3}\pi r^3 - \frac{2}{3}\pi(kr)^3}$ | M1<br>A1<br>M1 | $\Sigma mx/\Sigma m$ or moments<br>numerator<br>denominator | |
| | $= \frac{3}{8}r\left(\frac{1-k^4}{1-k^3}\right)$ | E1 | | |
| | | | | 4 |
| (iii) | $(1-\varepsilon)^3 = 1 - 3\varepsilon + 3\varepsilon^2 - \varepsilon^3$ | M1 | binomial expansion | |
| | $1 - k^3 \approx 1 - (1 - 3\varepsilon) = 3\varepsilon$ | E1 | | |
| | $1 - k^4 = 1 - (1 - 4\varepsilon + 6\varepsilon^2 - 4\varepsilon^3 + \varepsilon^4) \approx 4\varepsilon$ | A1 | | |
| | $\bar{x} = \frac{3}{8}r\left(\frac{1-k^4}{1-k^3}\right) \approx \frac{3}{8}r\left(\frac{4\varepsilon}{3\varepsilon}\right)$ | M1 | substituting | |
| | $= \frac{1}{2}r$ | E1 | must be all correct | |
| | | | | 5 |

# Examiner’s Report

## 2609 Mechanics 3

### General Comments

This was found to be an accessible paper with many candidates scoring well. There was a good level of understanding of the standard techniques, with the notable exception of showing that the motion of a system is simple harmonic. Some candidates lost marks through careless reading by solving the mechanics problems but omitting to give all the details asked for (for example the additional number of springs in question 2, the expression for tension in question 3 and the volume in question 4).

### Comments on Individual Questions

Q.1 Despite sloppy notation from some candidates (including overuse of square brackets such a $[M][L][T]^{-2}$ by a few candidates), most candidates were able to answer the majority of this question successfully. However the last part of the question was possibly the least well done in the whole paper. The serious attempts at this part generally tried to relate the formula for energy to the earlier dimensional work but few used the tension (in spite of the instruction given) and fully correct solutions were not common.

(i) $LT^{-1}, LT^{-2}, MLT^{-2}$, (iii) $ML^{-1}T^{-2}$, (iv) $1, \beta+\gamma=3$, (v) $1, 2, \frac{1}{2}$.

Q.2 Most candidates calculated the compressions correctly. Most candidates made a reasonable attempt at calculating the number of springs, however two common errors were to use the wrong weight and giving the total number of springs rather than the number of additional springs as requested. A minority of candidates were able to demonstrate SHM clearly but many others did not even attempt to use a Newton's second law equation. Of those who did start from an equation of motion, many omitted the weight and/or confused the displacement with the extension.

(i) 0.000245m, (ii) 0.003185m, (iii) 7, (iv) $9.24 \times 10^{-5}$.

Q.3 This question proved to be a good source of marks for many candidates. A common error was to fail to get the correct expression for radius in one or both of the two different cases. Another common slip was to omit the requested expression for tension in the first part of the question. Some candidates were unable to resolve correctly (some even managing to get a horizontal component of weight). Some candidates did not realise that for an 'equation of motion' a Newton's second law equation was required, instead giving a relationship between acceleration and velocity or angular speed.

(ii) $V \frac{3}{2}T = mg$, $\frac{1}{2}T = 3mw^2$, (iii) 1.37.

Q.4 The calculation of the centre of mass of the hemisphere was often well done, although some candidates omitted to explicitly calculate the volume as requested. Some also did not give a satisfactory explanation of the stated $y$-coordinate of the centre of mass - an appeal to symmetry was all that was required. The calculation of the centre of mass of the shell in part (ii) was often done well, but many candidates engaged in lengthy integration which was at best unnecessary but more often was incorrect. It was quite sufficient to replace $r$ by $kr$ in the results already established and then combine the results using moments. Of those who adopted the correct approach, many were successful but a common error was to use $kr^3$ rather than $(kr)^3$. The last part of the question was often done well, although some candidates seemed unsure what was required. In particular, showing the given approximation was sometimes lacking in detail or based on incorrect statements.

(i) $\frac{2}{3}\pi r^3$, (iii) $4\varepsilon$.