OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2609

Mechanics 3

Wednesday **21 JANUARY 2004** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take g = 9.8 m s^{-2} unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 4 printed pages.

[Turn over

1 A brass pendulum consists of a rod AB freely hinged at the end A with a sphere at the end B, as shown in Fig. 1.

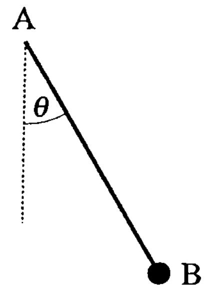


Fig. 1

When oscillating, the total energy, E, of the pendulum is given by the equation

$$E = \tfrac{1}{2}I\omega^2 - mgh\cos\theta,$$

where ω is the angular speed, m is the mass of the pendulum, h is the distance of the centre of mass of the pendulum from A, θ is the angle the pendulum makes with the downward vertical and I is a quantity known as the moment of inertia of the pendulum.

(i) Use this equation to deduce the dimensions of I. [4]

It is suggested that the period, T, of the pendulum is given by $T = kI^{\alpha}(mg)^{\beta}h^{\gamma}$ where k is a dimensionless constant.

(ii) Use dimensional analysis to find α, β and γ. [5]

For a particular pendulum, the equation of motion can be shown to be

$$\ddot{\theta} + 9\sin\theta = 0.$$

(iii) Show that, for small θ, the motion is approximately simple harmonic. [2]

The time, t seconds, is recorded from an instant when the pendulum is observed to be at an angle of 0.025 radians to the vertical and moving away from the vertical. The amplitude of the oscillations is observed to be 0.05 radians.

(iv) Find θ in terms of t and hence find the value of t when the pendulum first passes through the vertical. [4]

2 A railway engine of mass 50 tonnes travels at $30\,\text{m s}^{-1}$ along a section of horizontal track around a circular bend of radius 500 m.

(i) Calculate the lateral (sideways) force exerted on the rails by the engine, indicating clearly where you use Newton's second and third laws. [4]

Subsequently, for safety reasons, it is decided that the lateral force on the rails should not exceed 50 000 N. One way to ensure this is to impose a speed limit of $V\,\text{m s}^{-1}$, so that at this speed the force is 50 000 N.

(ii) Calculate V. [2]

Another way to restrict the force on the rails and allow the engine to travel at $30\,\text{m s}^{-1}$ is to bank the track at an angle θ to the horizontal. This is shown in Fig. 2, where AB is a line of greatest slope.

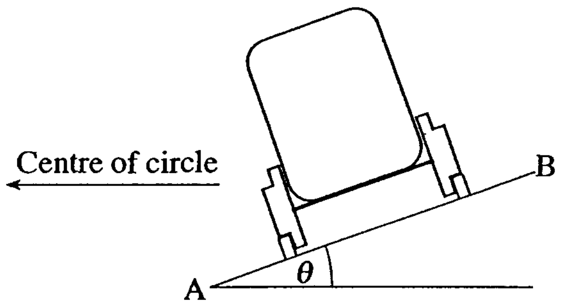


Fig. 2

(iii) When the engine travels at this speed, find, in terms of θ, the component of acceleration in the direction BA. Hence, or otherwise, show that the lateral force, T N, is given by

$$T = 90\,000\cos\theta - 490\,000\sin\theta. \quad [5]$$

(iv) Express T in the form $R\cos(\theta + \alpha)$. Hence calculate θ when $T = 50\,000$. [4]

Turn over for Questions 3 and 4.

3 A light elastic string AB, of natural length 0.8 m and modulus 5 N, is attached at A to a ceiling which is 2.4 m above the floor. A small ball of mass 0.2 kg is attached to the other end B and hangs in equilibrium.

(i) Calculate the length of the string. [3]

(ii) The ball is pulled down until it touches the floor with AB vertical and it is then released from rest. Calculate the speed at which it hits the ceiling. [4]

A second light elastic string of modulus 5 N and natural length l_1 m, where $l_1 < 2.4$, is attached to the ball at B and to the floor vertically below A. The ball is held at rest on the floor with AB vertical and it is then released.

(iii) Find the range of values of l_1 for which the ball will still hit the ceiling. [8]

4 A uniform lamina is in the shape of a triangle OAB, where O is the origin and A and B have coordinates (b, h) and $(b, 0)$ respectively, as shown in Fig. 4.

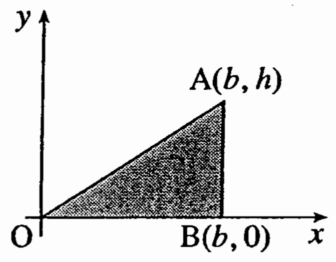


Fig. 4

(i) Find the equation of the line through O and A. Hence show by integration that the y-coordinate of the centre of mass of the lamina is $\frac{1}{3}h$.

Write down the x-coordinate of the centre of mass. [6]

A second triangular lamina, OBC, has vertices at O, B and the point C with coordinates (c, h) where $0 < c < b$.

(ii) Show that the centre of mass of OBC is at the point $(\frac{1}{3}(b + c), \frac{1}{3}h)$. [5]

A uniform triangular prism with cross-section OBC is placed on a rough plane inclined at 30° to the horizontal. The cross-section OBC is vertical with OB along a line of greatest slope and B higher than O.

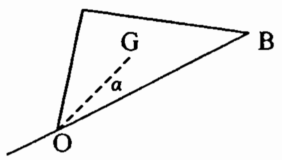
(iii) Find, in terms of b and h, the range of values of c for which the prism will topple. Deduce that the prism will not topple if $h < b\sqrt{3}$. [4]

Mark Scheme

1(i)	$[E](=[mgh\cos\theta]) = \text{ML}^2\text{T}^{-2}$	B1	stated or used correctly	
	$[\omega] = \text{T}^{-1}$	B1	stated or used correctly	
	$[I] = [E]/[\omega^2]$	M1		
	$= \text{ML}^2$	A1	from equation	4
(ii)	$\text{T} = (\text{ML}^2)^\alpha(\text{MLT}^{-2})^\beta\,\text{L}^\gamma$	M1	substitute dimensions (must be shown)	
	$\alpha + \beta = 0$	M1	compare indices for one dimension	
	$2\alpha + \beta + \gamma = 0$	M1	compare indices for all dimensions	
	$-2\beta = 1$			
	$\beta = -\frac{1}{2}$	A1	one correct	
	$\alpha = \frac{1}{2}, \gamma = -\frac{1}{2}$	A1	all correct if no comparison then M1 for substituting, A2 for one correct, A2 for all correct	5
(iii)	θ small $\Rightarrow \sin\theta \approx \theta$	B1		
	$\Rightarrow \ddot{\theta} \approx -9\theta \Rightarrow$ SHM	E1	must conclude SHM	2
(iv)	$\theta = 0.05\sin(3t + \varepsilon)$	M1	or equivalent (accept non-zero value for ε)	
	$0.025 = 0.05\sin\varepsilon \Rightarrow \varepsilon = \frac{1}{6}\pi$	M1	calculate ε	
	$\theta = 0.05\sin(3t + \frac{1}{6}\pi)$ or $\theta = 0.05\cos(3t - \frac{1}{3}\pi)$	A1		
	$\theta = 0 \Rightarrow 3t + \frac{1}{6}\pi = \pi \Rightarrow t = \frac{5}{18}\pi$	B1		4

2(i)	N2L towards centre: Force on engine $= 50000\dfrac{30^2}{500}$	M1 A1 A1	use of v^2/r identify as N2L all correct	
	N3L: Force on rails = Force on engine = 90000	B1	identify both forces and use of N3L	4
(ii)	$50000 = 50000\dfrac{V^2}{500}$	M1	N2L with v^2/r	
	$\Rightarrow V = 10\sqrt{5} \approx 22.4\text{ m s}^{-1}$	A1		2
(iii)	component of acceleration $= \dfrac{30^2}{500}\cos\theta$	B1		
	$T + 50000g\sin\theta = 50000\dfrac{30^2}{500}\cos\theta$	B1 M1 A1	resultant force down bank N2L with component of acceleration correct equation (not $T = \ldots$)	
	$T = 90000\cos\theta - 490000\sin\theta$	E1	must follow from correct N2L equation	
	Alternative:			
	$T\cos\theta + R\sin\theta = 50000\dfrac{30^2}{500}$	B1		
	$R\cos\theta = T\sin\theta + 50000g$	B1		
	eliminate R	M1		
	$T = 90000\cos\theta - 490000\sin\theta$	E1		5
(iv)	$R = \sqrt{90000^2 + 490000^2} \approx 498200$	B1		
	$\alpha = \tan^{-1}(490000/90000) \approx 79.59$	M1		
	$R\cos(\theta + \alpha) = 50000 \Rightarrow \theta + \alpha = \cos^{-1}(50000/R) = 84.24$	M1	solving	
	$\Rightarrow \theta \approx 4.65°$	A1		4

3(i)	$0.2g = 5x/0.8$	M1	use of Hooke's law	
	$x = 0.3136$	A1		
	length = 1.11(36) m	A1		3
(ii)	$\frac{1}{2}0.2v^2 + 0.2g \times 2.4 = \frac{5 \times 1.6^2}{2 \times 0.8}$	M1	use of energy	
		A1	EPE term	
		A1	correct equation	
	$\Rightarrow v \approx 5.74 \text{ m s}^{-1}$	A1		4
(iii)	$\frac{1}{2}0.2v^2 + 0.2g \times 2.4 + \frac{5(2.4 - l_1)^2}{2l_1} = \frac{5 \times 1.6^2}{2 \times 0.8}$	M1	attempt EPE in terms of l_1	
		A1	correct EPE in terms of l_1	
		M1	energy equation (everything considered)	
		A1	all correct	
	$v^2 \geq 0 \Rightarrow$	M1	use of $v^2 \geq 0$ (accept =)	
	$l_1^2 - 6.1184l_1 + 5.76 \leq 0$	A1	equation or inequality (or multiple)	
	$\Rightarrow 1.162 \leq l_1 \leq 4.956$	M1	solving quadratic	
	$l_1 \geq 1.162$	A1	inequality must be justified by algebra or explanation	8

4(i)	$y = \frac{h}{b}x$	B1		
	$\frac{1}{2}bh\bar{y} = \int_0^b \frac{1}{2}\left(\frac{h}{b}x\right)^2 dx$	B1	formula stated or used correctly	
	$= \frac{h^2}{2b^2}\left[\frac{1}{3}x^3\right]_0^b = \frac{1}{6}h^2 b$	M1	substitute and integrate RHS	
		M1	use correct limits and divide by area	
	$\bar{y} = \frac{1}{3}h$	E1		
	$\bar{x} = \frac{2}{3}b$	B1		6
(ii)	shear $\Rightarrow$ mass distribution from x-axis unchanged	M1	or calculation	
	$\Rightarrow \bar{y} = \frac{1}{3}h$	E1		
	$\frac{1}{2}bh\bar{x} = \frac{1}{2}ch \cdot \frac{2}{3}c + \frac{1}{2}(b-c)h \cdot (c + \frac{1}{3}(b-c))$	M1	reasonable attempt at $\bar{x}$	
		A1	correct equation	
	$\bar{x} = \frac{1}{3}b + \frac{1}{3}c$	E1		5
(iii)				
	topples $\Leftrightarrow \alpha > 60°$	B1	(accept =)	
	($c > 0 \Rightarrow \alpha$ acute) so topples $\Leftrightarrow \tan\alpha > \sqrt{3}$	M1	(accept =)	
	$\Leftrightarrow \frac{\frac{1}{3}h}{\frac{1}{3}b + \frac{1}{3}c} > \sqrt{3} \Leftrightarrow c < \frac{h}{\sqrt{3}} - b$	A1	inequality must be justified by algebra or explanation	
	as $c > 0$, $h < \sqrt{3}b \Rightarrow c > \frac{h}{\sqrt{3}} - b \Rightarrow$ does not topple	E1		4

Examiner’s Report

2609 Mechanics 3

General Comments

Several centres expressed concern about the difficulty and length of this paper and many candidates clearly found the latter parts of the paper hard. These points were accepted and allowance was made when setting the grade boundaries. It is also greatly regretted that question 2(iv) required a technique from P3 which is not assumed knowledge for the paper. Again, allowance was made when setting the grade boundaries and, in addition, the work on this part of every candidate was reviewed.

Comments on Individual Questions

Question 1 (Dimensions and simple harmonic motion)

The dimensions parts of this question were often well answered, except that many candidates did not know the dimensions of angular speed. A surprising number of candidates did not know how to use the small angle approximation to deduce the simple harmonic motion equation for the pendulum. Many candidates tried to use the solution of the equation in the form $\theta = a\cos\omega t$, contrary to the given initial conditions.

(i) ML^2; (ii) $\alpha = \frac{1}{2}, \beta = \gamma = -\frac{1}{2}$; (iv) $\theta = 0.05\sin(3t + \frac{1}{6}\pi), t = \frac{5}{18}\pi$

Question 2 (Circular motion)

In the first part, the calculation using Newton's second law was usually correct, but many candidates did not identify the use of the law as requested. Many candidates were unable to recall Newton's third law correctly, and even those who did often did not make it clear how it was used in this calculation. The calculation of maximum speed was usually correct. When finding the expression for *T*, candidates who used the suggestion in the question to use the component of acceleration often gave short and clear solutions. Those who preferred to work horizontally and vertically often produced longer but good solutions, although some omitted the lateral force from their vertical equation. However, many candidates provided insufficient working to justify where their result came from. Many solutions began *T* = ..., without any justification for the terms or signs in the expression given. When showing a given result, candidates are best advised to set up their Newton's second law equation in the form 'resultant force equals mass multiplied by acceleration'.

(i) 90 000 N; (ii) 22.4 m s^{-1}; (iv) 4.65°

Question 3 (Elastic strings)

Most candidates were able to calculate the length of the string accurately. Many candidates were able to calculate the velocity using energy, but some omitted the gravitational potential energy. Some candidates split the motion into stages and tried to use methods other than energy. These attempts always took longer and were rarely correct. In the last part, many candidates recognised the need to use energy, but most were unable to set up an accurate energy equation.

(i) 1.11 m; (ii) 5.74 m s^{-1}; (iii) $\ell_1 \geqslant 1.162$

Question 4 (Centre of mass)

Candidates generally were able to use integration to find the required result in the first part of the question. Some were able accurately to use this result or an alternative method to calculate the centre of mass in the second part. Few candidates were able correctly to identify when the prism toppled.

(i) $\bar{x} = \frac{2}{3}b$; (iii) $c < \frac{h}{\sqrt{3}} - b$