4 i)
$$e^{jk\theta} = \cos k\theta + j\sin k\theta$$
$$e^{-jk\theta} = \cos k\theta - j\sin k\theta$$

$$e^{2j\theta} - 1 = e^{j\theta}\left(e^{j\theta} - e^{-j\theta}\right) = e^{j\theta}\left(\cos\theta + j\sin\theta - (\cos\theta - j\sin\theta)\right)$$
$$= e^{j\theta}(2j\sin\theta)$$
$$= 2je^{j\theta}\sin\theta$$

ii)
$$C = \cos\theta + \cos 3\theta + \cos 5\theta + \dots + \cos(2n-1)\theta$$
$$S = \sin\theta + \sin 3\theta + \sin 5\theta + \dots + \sin(2n-1)\theta$$
for $0 < \theta < \frac{\pi}{n}$

$$C + jS = (\cos\theta + j\sin\theta) + (\cos 3\theta + j\sin 3\theta) + \dots + (\cos(2n-1)\theta + j\sin(2n-1)\theta)$$
$$= e^{j\theta} + e^{j3\theta} + e^{j5\theta} + \dots + e^{j(2n-1)\theta}$$

This is a GP with 1st term $a = e^{j\theta}$

common ratio $r = e^{j2\theta}$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{e^{j\theta}(e^{j2n\theta} - 1)}{e^{j2\theta} - 1}$$

iii)
$$|C + jS| = \left|\frac{e^{j\theta}(e^{j2n\theta} - 1)}{e^{j2\theta} - 1}\right| = \left|\frac{\cancel{e^{j\theta}}\, e^{jn\theta}(\cancel{2}j\sin n\theta)}{\cancel{2}j\cancel{e^{j\theta}}\sin\theta}\right|$$
$$= \left|\frac{e^{jn\theta}\sin n\theta}{\sin\theta}\right|$$

4 iii) cont)

$$|C+jS| = |e^{jn\theta}|\left|\frac{\sin n\theta}{\sin\theta}\right|$$

$$= \left|\frac{\sin n\theta}{\sin\theta}\right| \quad \text{since } |e^{jn\theta}| = 1$$

$$= \frac{\sin n\theta}{\sin\theta} \quad \text{since both positive for } 0<\theta<\frac{\pi}{n}$$

$$\arg(C+jS) = n\theta$$

This because $C+jS = e^{jn\theta}\dfrac{\sin n\theta}{\sin\theta}$ from previous page

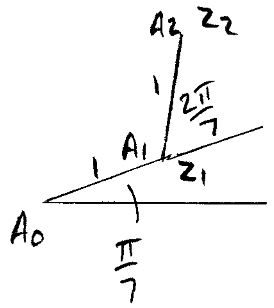
iv)

$$C+jS = \frac{\sin n\theta}{\sin\theta}\left(\cos n\theta + j\sin n\theta\right)$$

Equating real and imaginary parts

$$C = \frac{\sin n\theta \cos n\theta}{\sin\theta}, \quad S = \frac{\sin^2 n\theta}{\sin\theta}$$

v)



$$z_2 - z_1 = e^{j\frac{3\pi}{7}} \quad \left(\text{modulus } 1, \ \tfrac{3\pi}{7} \text{ with real axis}\right)$$

$$z_3 - z_2 = e^{j\frac{5\pi}{7}} \quad \text{etc}$$

$$\Rightarrow z_2 = z_1 + e^{j\frac{3\pi}{7}}$$

$$= e^{j\frac{\pi}{7}} + e^{j\frac{3\pi}{7}}$$

$$\Rightarrow z_3 = e^{j\frac{5\pi}{7}} + z_2 = e^{j\frac{\pi}{7}} + e^{j\frac{3\pi}{7}} + e^{j\frac{5\pi}{7}}$$

and so on to $z_6 = e^{j\frac{\pi}{7}} + e^{j\frac{3\pi}{7}} + \ldots + e^{j\frac{11\pi}{7}}$

From part ii $\quad z_n = C + jS \quad$ with $\theta = \frac{\pi}{7}$

$\therefore$ from part iii $\quad \arg z_n = \arg(C+jS) = n\theta = \dfrac{n\pi}{7}$