FP2 PURE 5 JANUARY 2002 Q2 COMPLEX NUMBERS

2 a)

By de Moivre's theorem

$$\cos 5\theta + j\sin 5\theta = (\cos\theta + j\sin\theta)^5$$

Let $c = \cos\theta$, $s = \sin\theta$

$$= (c+js)^5 = c^5 + 5c^4js + 10c^3j^2s^2 + 10c^2j^3s^3 + 5cj^4s^4 + j^5s^5$$

$$= c^5 + 5c^4js - 10c^3s^2 - 10c^2js^3 + 5cs^4 + js^5$$

Equating real parts

$$\cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$$

$$= c^5 - 10c^3(1-c^2) + 5c(1-c^2)^2$$

$$= c^5 - 10c^3 + 10c^5 + 5c(1-2c^2+c^4)$$

$$= c^5 - 10c^3 + 10c^5 + 5c - 10c^3 + 5c^5$$

$$= 16c^5 - 20c^3 + 5c$$

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

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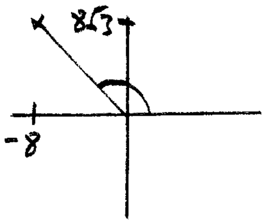
b) i)

$$|-8+8\sqrt{3}j| = \sqrt{(-8)^2 + (8\sqrt{3})^2} = \sqrt{64+192}$$

$$= \sqrt{256} = 16$$



$$\arg(-8+8\sqrt{3}j) = \pi - \tan^{-1}\frac{8\sqrt{3}}{8}$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

$$|-8+8\sqrt{3}j| = 16 \qquad \arg(-8+8\sqrt{3}j) = \frac{2\pi}{3}$$

2bi) cont) $\Rightarrow \quad -8 + 8\sqrt{3}j = 16e^{j\frac{2\pi}{3}}$

fourth roots given by $16^{\frac{1}{4}} e^{j\left(\frac{2\pi}{12} + \frac{2n\pi}{4}\right)}$ for $n = 0, 1, 2, 3$

$= 2e^{j\frac{\pi}{6}}, \quad 2e^{j\frac{2\pi}{3}}, \quad 2e^{j\frac{7\pi}{6}}, \quad 2e^{j\frac{10\pi}{6}}$

$= 2e^{j\frac{\pi}{6}}, \quad 2e^{j\frac{2\pi}{3}}, \quad 2e^{-j\frac{5\pi}{6}}, \quad 2e^{-j\frac{\pi}{3}}$

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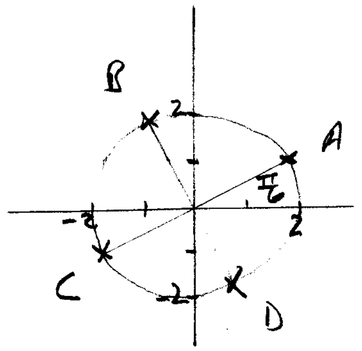
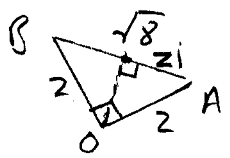
ii)





Side of square $= \sqrt{8}$

or $2\sqrt{2}$

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iii) $\quad OZ_1 = \dfrac{\sqrt{8}}{2} = \dfrac{2\sqrt{2}}{2} = \sqrt{2}$ (isos Δ with 45° angles)

$\Rightarrow \quad |z_1| = \sqrt{2} \qquad \arg z_1 = \frac{\pi}{6} + \frac{1}{2} \times \frac{\pi}{2}$

$= \frac{\pi}{6} + \frac{\pi}{4} = \frac{5\pi}{12}$

$w = z_1^4 = (\sqrt{2})^4 e^{j\left(4 \times \frac{5\pi}{12}\right)}$

$= 4e^{j\frac{5\pi}{3}}$

$= 4\left(\cos\frac{5\pi}{3} + j\sin\frac{5\pi}{3}\right)$

$= 4\left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)$

$w = 2 - 2\sqrt{3}j$
