FP2 PURE 5 JANUARY 2004 Q2 COMPLEX NUMBERS

2 i) $z^n + \frac{1}{z^n} = 2\cos(n\theta)$, $z^n - \frac{1}{z^n} = 2j\sin(n\theta)$

ii) $\left(z - \frac{1}{z}\right)^6 = (2j\sin\theta)^6 = -64\sin^6\theta$

$\left(z - \frac{1}{z}\right)^6 = z^6 - 6z^5\frac{1}{z} + 15z^4\frac{1}{z^2} - 20z^3\frac{1}{z^3} + 15z^2\frac{1}{z^4} - 6z\frac{1}{z^5} + \frac{1}{z^6}$

$= \left(z^6 + \frac{1}{z^6}\right) - 6\left(z^4 + \frac{1}{z^4}\right) + 15\left(z^2 + \frac{1}{z^2}\right) - 20$

$= 2\cos6\theta - 12\cos4\theta + 30\cos2\theta - 20$

$\Rightarrow \sin^6\theta = \dfrac{2\cos6\theta - 12\cos4\theta + 30\cos2\theta - 20}{-64}$

$\sin^6\theta = -\frac{1}{32}\cos6\theta + \frac{3}{16}\cos4\theta - \frac{15}{32}\cos2\theta + \frac{5}{16}$

iii) $Re\left(e^{x+yj}\right) = Re\left(e^x e^{yj}\right) = Re\left(e^x(\cos y + j\sin y)\right)$

$= e^x\cos y$

$Im\left(e^{x+yj}\right) = Im\left(e^x e^{yj}\right) = Im\left(e^x(\cos y + j\sin y)\right)$

$= e^x\sin y$

iv) $e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + +$

2v) $$C = 1 + \cos\theta + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} +$$

$$S = \sin\theta + \frac{\sin 2\theta}{2!} + \frac{\sin 3\theta}{3!}$$

$$C + jS = 1 + (\cos\theta + j\sin\theta) + \frac{(\cos 2\theta + j\sin 2\theta)}{2!} + \frac{(\cos 3\theta + j\sin 3\theta)}{3!} +$$

$$= 1 + e^{j\theta} + \frac{e^{j2\theta}}{2!} + \frac{e^{j3\theta}}{3!} + \ldots$$

$$= e^{e^{j\theta}}$$

$$= e^{\cos\theta + j\sin\theta}$$

From iii)

$$C = \mathrm{Re}\left(e^{\cos\theta + j\sin\theta}\right) = e^{\cos\theta}\cos(\sin\theta)$$

$$S = \mathrm{Im}\left(e^{\cos\theta + j\sin\theta}\right) = e^{\cos\theta}\sin(\sin\theta)$$