3a) $z^5 = (\cos\theta + j\sin\theta)^5 = (\cos 5\theta + j\sin 5\theta)$

Letting $c = \cos\theta$, $s = \sin\theta$

$$z^5 = (c + js)^5$$

$$= c^5 + 5c^4 js + 10c^3 j^2 s^2 + 10c^2 j^3 s^3 + 5c j^4 s^4 + j^5 s^5$$

$$= c^5 + 5c^4 js - 10c^3 s^2 - 10c^2 js^3 + 5cs^4 + js^5$$

Equating real and imaginary parts

$$\cos 5\theta = c^5 - 10c^3 s^2 + 5cs^4$$

$$\sin 5\theta = 5c^4 s - 10c^2 s^3 + s^5$$

$$\tan 5\theta = \frac{\sin 5\theta}{\cos 5\theta} = \frac{5c^4 s - 10c^2 s^3 + s^5}{c^5 - 10c^3 s^2 + 5cs^4}$$

Divide numerator and denominator by c^5

$$\tan 5\theta = \frac{\frac{5c^4 s}{c^5} - \frac{10c^2 s^3}{c^5} + \frac{s^5}{c^5}}{\frac{c^5}{c^5} - \frac{10c^3 s^2}{c^5} + \frac{5cs^4}{c^5}}$$

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$$

3b)

i) $z^n + \frac{1}{z^n} = 2\cos(n\theta)$, $z^n - \frac{1}{z^n} = 2j\sin(n\theta)$

ii) $$\left[\left(z - \frac{1}{z}\right)\left(z + \frac{1}{z}\right)\right]^3 = \left[2j\sin\theta \times 2\cos\theta\right]^3$$

$$= -64j\sin^3\theta\cos^3\theta$$

But also

$$\left[\left(z - \frac{1}{z}\right)\left(z + \frac{1}{z}\right)\right]^3 = \left(z - \frac{1}{z}\right)^3\left(z + \frac{1}{z}\right)^3$$

$$= \left(z^3 - 3z^2\frac{1}{z} + 3z\frac{1}{z^2} - \frac{1}{z^3}\right)\left(z^3 + 3z^2\frac{1}{z} + 3z\frac{1}{z^2} + \frac{1}{z^3}\right)$$

$$= \left(z^3 - 3z + \frac{3}{z} - \frac{1}{z^3}\right)\left(z^3 + 3z + \frac{3}{z} + \frac{1}{z^3}\right)$$

$$\begin{aligned} = \; & z^6 - 3z^4 + 3z^2 - 1 \\ & + 3z^4 - 9z^2 + 9 - \frac{3}{z^2} \\ & + 3z^2 - 9 + \frac{9}{z^2} - \frac{3}{z^4} \\ & + 1 - \frac{3}{z^2} + \frac{3}{z^4} - \frac{1}{z^6} \end{aligned}$$

$$= z^6 - 3z^2 + \frac{3}{z^2} - \frac{1}{z^6}$$

$$= \left(z^6 - \frac{1}{z^6}\right) - 3\left(z^2 - \frac{1}{z^2}\right)$$

3bii) cont

$$= 2j\sin 6\theta - 6j\sin 2\theta$$

$$\Rightarrow -64j\sin^3\theta\cos^3\theta = 2j\sin 6\theta - 6j\sin 2\theta$$

$$\Rightarrow \sin^3\theta\cos^3\theta = \frac{-\sin 6\theta + 3\sin 2\theta}{32}$$

$$\Rightarrow \sin^3\theta\cos^3\theta = \frac{3}{32}\sin 2\theta - \frac{1}{32}\sin 6\theta$$

3biii)

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - + \ldots$$

$$\sin^3\theta\cos^3\theta = \frac{3}{32}\left[2\theta - \frac{(2\theta)^3}{3!} + \frac{(2\theta)^5}{5!} -\right] - \frac{1}{32}\left[6\theta - \frac{(6\theta)^3}{3!} + \frac{(6\theta)^5}{5!} -\right]$$

$$\approx \frac{3}{32}\left[2\theta - \frac{8\theta^3}{6} + \frac{32\theta^5}{120}\right] - \frac{1}{32}\left[6\theta - 36\theta^3 + \frac{7776\theta}{120}\right]$$

$$\approx \theta^3 - 2\theta^5$$

3biv) If θ^7 and higher powers can be neglected

$$\sin^3\theta\cos^3\theta \approx \theta^3(1 - 2\theta^2)$$

$$= \theta^3\left(1 - \frac{(2\theta)^2}{2}\right)$$

$$\approx \theta^3\cos 2\theta$$
