FP2 PURE 5 JUNE 2003 Q3 COMPLEX NUMBERS

3 i)

$$e^{-\frac{1}{2}j\theta} - e^{\frac{1}{2}j\theta} = -2j\sin\frac{\theta}{2}$$

$$(1-e^{j\theta})^2 = \left(e^{\frac{1}{2}j\theta}\left(e^{-\frac{1}{2}j\theta} - e^{\frac{1}{2}j\theta}\right)\right)^2$$

$$= \left(e^{\frac{1}{2}j\theta}\right)^2\left(-2j\sin\frac{\theta}{2}\right)^2$$

$$= e^{j\theta}\left(4j^2\sin^2\frac{\theta}{2}\right)$$

$$= -4e^{j\theta}\sin^2\frac{\theta}{2}$$

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3 ii)

$$C = 1 - \binom{2n}{1}\cos\theta + \binom{2n}{2}\cos 2\theta - \dots + \binom{2n}{2n}\cos 2n\theta$$

$$S = -\binom{2n}{1}\sin\theta + \binom{2n}{2}\sin 2\theta - \dots + \binom{2n}{2n}\sin 2n\theta$$

$$C+jS = 1 - \binom{2n}{1}e^{j\theta} + \binom{2n}{2}e^{2j\theta} - \binom{2n}{3}e^{3j\theta} + \dots + \binom{2n}{2n}e^{2nj\theta}$$

$$= \left(1-e^{j\theta}\right)^{2n}$$

Using part (i)

$$C+jS = \left(-4e^{j\theta}\sin^2\left(\frac{\theta}{2}\right)\right)^n$$

$$= (-4)^n e^{jn\theta}\sin^{2n}\frac{\theta}{2}$$

$$= (-4)^n\left(\cos n\theta + j\sin n\theta\right)\sin^{2n}\frac{\theta}{2}$$

3ii) cont) Equating real and imaginary parts:

$$C = (-4)^n \cos n\theta \sin^{2n}\left(\frac{\theta}{2}\right)$$

$$S = (-4)^n \sin n\theta \sin^{2n}\left(\frac{\theta}{2}\right)$$

3iii) $w = e^{j\phi}$ with $w^3 = 1$

$$\Rightarrow \phi = 0, \frac{2\pi}{3}, -\frac{2\pi}{3}$$

When $\phi = 0$,

$$w = 1 \quad \Rightarrow (1-w)^6 = 0$$

When $\phi = \frac{2\pi}{3}$

$$(1-w)^6 = \left(1 - e^{j\frac{2\pi}{3}}\right)^6 = \left(-4e^{j\frac{2\pi}{3}}\sin^2\frac{\pi}{3}\right)^3$$

$$= -64\, e^{j2\pi} \sin^6\frac{\pi}{3}$$

$$= -64 \times \left(\frac{\sqrt{3}}{2}\right)^6 = -27$$

When $\phi = -\frac{2\pi}{3}$ $\quad (1-w)^6 = \left(-4\, e^{-j\frac{2\pi}{3}}\sin^2\left(-\frac{\pi}{3}\right)\right)^3$

$$= -64\, e^{-j2\pi} \sin^6\left(-\frac{\pi}{3}\right)$$

$$= -64 \times \left(-\frac{\sqrt{3}}{2}\right)^6 = -27$$

$$\therefore (1-w)^6 = 0 \text{ or } -27$$