3) a)

$$\cos 4\theta + j\sin 4\theta = (c + js)^4 \qquad \text{where } c = \cos\theta,\ s = \sin\theta$$

$$= c^4 + 4c^3js + 6c^2j^2s^2 + 4cj^3s^3 + j^4s^4$$

$$= c^4 + 4c^3js - 6c^2s^2 - 4cjs^3 + s^4$$

Equating real and imaginary parts

$$\cos 4\theta = c^4 - 6c^2s^2 + s^4$$

$$\sin 4\theta = 4c^3s - 4cs^3$$

$$\tan 4\theta = \frac{\sin 4\theta}{\cos 4\theta} = \frac{4c^3s - 4cs^3}{c^4 - 6c^2s^2 + s^4}$$

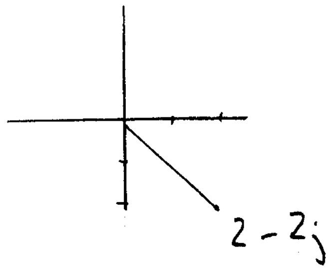
$\div c^4$ numerator and denominator

$$\tan 4\theta = \frac{\frac{4c^3s}{c^4} - \frac{4cs^3}{c^4}}{\frac{c^4}{c^4} - \frac{6c^2s^2}{c^4} + \frac{s^4}{c^4}}$$

$$\tan 4\theta = \frac{\frac{4s}{c} - \frac{4s^3}{c^3}}{1 - \frac{6s^2}{c^2} + \frac{s^4}{c^4}}$$

$$\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$$

3b) i)



$$|2-2j| = \sqrt{2^2+(-2)^2} = \sqrt{8} = 2\sqrt{2}$$

$$\arg(2-2j) = -\frac{\pi}{4}$$

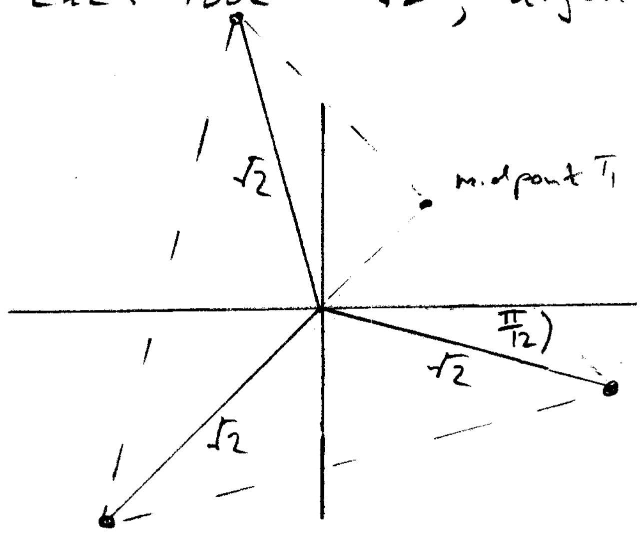
ii)

cube roots of $2-2j$ given by

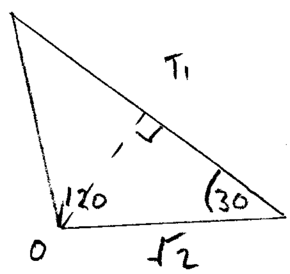
$$(2\sqrt{2})^{\frac{1}{3}} e^{j\left(-\frac{\pi}{12}+\frac{2n\pi}{3}\right)} \qquad n = 0, 1, 2$$

$$= \sqrt{2}\, e^{-j\frac{\pi}{12}}, \quad \sqrt{2}\, e^{j\frac{7\pi}{12}}, \quad \sqrt{2}\, e^{-j\frac{3\pi}{4}}$$

Modulus of each root $= \sqrt{2}$, arguments $-\frac{\pi}{12}, \frac{7\pi}{12}, -\frac{3\pi}{4}$



iii)



$$\frac{|OT_1|}{\sqrt{2}} = \sin 30 \qquad |OT_1| = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

Modulus of each midpoint $= \frac{1}{\sqrt{2}}$

arguments $\dfrac{-\frac{\pi}{12}+\frac{7\pi}{12}}{2} = \frac{\pi}{4}$

Also $\frac{\pi}{4}+\frac{2\pi}{3} = \frac{11\pi}{12}$, $\frac{\pi}{4}-\frac{2\pi}{3} = -\frac{5\pi}{12}$

3iii) cont) Modulus of each midpoint $= \frac{1}{\sqrt{2}}$

Arguments $\frac{\pi}{4}$, $\frac{11\pi}{12}$, $-\frac{5\pi}{12}$

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iv) consider $\frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}}$

$$w = \left(\frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}}\right)^3$$

$$w = \left(\frac{1}{\sqrt{2}}\right)^3 e^{j\frac{3\pi}{4}}$$

$$w = \frac{1}{2\sqrt{2}}\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right)$$

$$w = \frac{1}{2\sqrt{2}}\left(-\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$$

$$w = -\frac{1}{4} + \frac{1}{4}j$$