3 i) A) $e^{j\theta} + e^{-j\theta} = 2\cos\theta$

B) $(1-3e^{j\theta})(1-3e^{-j\theta})$

$= 1 - 3e^{j\theta} - 3e^{-j\theta} + 9$

$= 10 - 3(e^{j\theta} + e^{-j\theta})$

$= 10 - 6\cos\theta$

---

3 ii)

$$C = \cos\theta + 3\cos 2\theta + 9\cos 3\theta + \dots + 3^{n-1}\cos(n\theta)$$

$$S = \sin\theta + 3\sin 2\theta + 9\sin 3\theta + \dots + 3^{n-1}\sin(n\theta)$$

$$C+jS = (\cos\theta + j\sin\theta) + 3(\cos 2\theta + j\sin 2\theta) + \dots 3^{n-1}(\cos n\theta + j\sin n\theta)$$

$$C+jS = e^{j\theta} + 3e^{j2\theta} + 9e^{j3\theta} + \dots + 3^{n-1}e^{jn\theta}$$

This is a GP first term $a = e^{j\theta}$

common ratio $r = 3e^{j\theta}$

$$S_n = \frac{a(r^n-1)}{r-1} = \frac{a(1-r^n)}{1-r}$$

$$\Rightarrow \quad C+jS = \frac{e^{j\theta}\left(1-(3e^{j\theta})^n\right)}{1-3e^{j\theta}}$$

$$\Rightarrow \quad C+jS = \frac{e^{j\theta}(1-3^n e^{jn\theta})(1-3e^{-j\theta})}{(1-3e^{j\theta})(1-3e^{-j\theta})}$$

3ii cont)

$$\Rightarrow C + jS = \frac{e^{j\theta}\left(1 - 3^n e^{jn\theta} - 3e^{-j\theta} + 3^{n+1} e^{j(n-1)\theta}\right)}{10 - 6\cos\theta}$$

$$\Rightarrow C + jS = \frac{e^{j\theta} - 3^n e^{j(n+1)\theta} - 3 + 3^{n+1} e^{jn\theta}}{10 - 6\cos\theta}$$

$$\Rightarrow C + jS = \frac{(\cos\theta + j\sin\theta) - 3^n\left(\cos(n+1)\theta + j\sin(n+1)\theta\right) - 3 + 3^{n+1}(\cos n\theta + j\sin n\theta)}{10 - 6\cos\theta}$$

Equating real and imaginary parts

$$C = \frac{\cos\theta - 3^n\cos(n+1)\theta - 3 + 3^{n+1}\cos n\theta}{10 - 6\cos\theta}$$

$$S = \frac{\sin\theta - 3^n\sin(n+1)\theta + 3^{n+1}\sin n\theta}{10 - 6\cos\theta}$$

3iii)

$$27j = 27e^{j\frac{\pi}{2}}$$

Cubic roots are $27^{\frac{1}{3}} e^{j\left(\frac{\pi}{6} + \frac{2n\pi}{3}\right)}$ $\quad n = 0, 1, 2$

$$= 3e^{j\frac{\pi}{6}},\ 3e^{j\frac{5\pi}{6}},\ 3e^{-j\frac{\pi}{2}}$$

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3iv) Find possible values of $(1-w)(1-w^*)$

when $w^3 = 27j$

Using part (i) result $(1-3e^{j\theta})(1-3e^{-j\theta}) = 10 - 6\cos\theta$

When $w = 3e^{j\frac{\pi}{6}}$ $(1-w)(1-w^*) = 10 - 6\cos\frac{\pi}{6}$

$= 10 - 6 \times \frac{\sqrt{3}}{2} \quad = 10 - 3\sqrt{3}$

when $w = 3e^{j\frac{5\pi}{6}}$ $(1-w)(1-w^*) = 10 - 6\cos\frac{5\pi}{6}$

$= 10 + 6 \times \frac{\sqrt{3}}{2} \quad = 10 + 3\sqrt{3}$

when $w = 3e^{-j\frac{\pi}{2}}$ $(1-w)(1-w^*) = 10 - 6\cos(-\frac{\pi}{2})$

$= 10 - 0$

$= 10$