Statistics 2

Samples and hypothesis testing

Chapter assessment

1. A factory manager is specifying a new storage tank for a particular chemical. In routine use, the tank will be filled to capacity each weekend. There should be enough chemical to last until the next weekend, as emergency deliveries are very expensive. On the other hand, money is wasted if an excessive amount of the chemical is stored.

   The volume of chemical varies from week to week and is modelled by a Normally distributed random variable X. The manager is investigating the mean of X. Data are available for a random sample of 15 weeks, giving the volumes of the chemical used in each week. These are as follows (in litres).

1962	1928	1943	1939	1866	1964	1942	1996
1909	1940	1897	1924	1978	1944	1992	

   The standard deviation of X is taken from long experience to be 28 litres.

   A 2000-litre tank will be specified if the mean of X is no more than 1930 litres. Carry out a 5% significance test to examine whether a 2000-litre tank should be specified, stating clearly the null and alternative hypotheses and the conclusion.
   [8]

2. A craftsman makes hand-made souvenirs. The time taken to make a souvenir is a Normally distributed random variable with mean 34 minutes and standard deviation 2.6 minutes.

   The craftsman undertakes a training course to improve his skill. Afterwards, a random sample of 8 times taken to make souvenirs is as follows (in minutes).

35.4	32.3	26.6	30.4	31.9	33.8	29.6	28.4

   Assuming that the underlying standard deviation has not changed, test at the 0.1% level whether the mean time taken to make a souvenir has decreased after the training course.
   [8]

3. Psychologists are developing a new index of overall intelligence for 11-year old children. It is assumed that the index is Normally distributed over the whole underlying population and that the standard deviation of this distribution is 12. If the index has been created correctly, its mean over the population should be 50.

   The index is measured for a random sample of 100 11-year old children. It is found that the sample mean value is 47.8. Test the hypothesis that the true mean of the index is 50, against the alternative that it is not 50, at the 1% level of significance.
   [8]

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4. An office experiences a lack of reliability of its email system when transmitting messages. Many emails are successfully transmitted at the first attempt, others are eventually successfully transmitted, but only after more than one attempt, and others are not successfully transmitted at all. The computer manager thinks there may be an association between the success of transmission and the type of user at the intended destination. Results for a random sample of 300 emails are as follows.

		Type of destination		
		Commercial user	Government department	University
Transmission	Successful at first attempt	100	57	23
	Successful after more than one attempt	21	14	13
	Not successful at all	31	21	20

(i)   State the null and alternative hypotheses under examination in the usual χ^2 test applied to this contingency table.   [2]

(ii) Carry out the test, at the 10% significance level.   [12]

(iii) Discuss your conclusions.   [4]

5. As part of a survey of interest in local elections, a random sample of 845 people was taken in towns which did not have directly-elected mayors. The people were classified according to age (< 30 or ≥ 30) and their stated level of interest (great or little) in local elections. The results were as follows.

		Level of interest	
		Great	Little
Age	< 30	49	216
	≥ 30	145	435

(i)   Carry out the usual χ^2 test for independence, at the 5% significance level, stating carefully the null and alternative hypotheses and briefly discussing the conclusions.   [12]

At a later stage in the survey, a random sample of 1327 people was taken in towns which had directly elected mayors. These people were classified similarly, with the following results.

		Level of interest	
		Great	Little
Age	< 30	118	314
	≥ 30	260	635

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(ii) The organisation that commissioned the survey then asked whether, for people under the age of 30, the level of interest in local elections is independent of whether or not there is a locally elected mayor. Using the data in the tables above, write down the 2×2 table, including its margins, to be analysed. [4]

(iii) Explain why the usual χ^2 test might not be appropriate for the 2×2 table you have written down in part (ii). [2]

Total 60